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PRIME KNOTS WITH ARC INDEX 12 UP TO 16 CROSSINGS

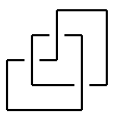
Gyo Taek Jin, Hyuntae Kim, Seungwoo Lee, and Hun Joo Myung

Abstract. We obtain the list of prime knots with arc index 12 up to 16 crossings and their minimal grid diagrams. This is a continuation of the works [5] and [8] in which Cromwell matrices were generated to obtain minimal grid diagrams of all prime knots up to arc index 11. We provide minimal grid diagrams of the prime alternating knots with arc index 12. They are the 10 crossing prime alternating knots. The full list of 19,513 prime knots of arc index 12 up to 16 crossings and their minimal grid diagrams can be found in the arXiv [6].

1. Introduction

An *arc presentation* of a knot is an embedding of a knot into the union of finitely many vertical half planes whose common boundary is the z-axis so that each of the half planes intersect the knot in a single properly embedded curve. These curves are called the *arcs* of the arc presentation. The minimal number of arcs among all arc presentations of a given knot is called the *arc index* [1,2].

A *grid diagram* is a knot diagram whose projection is a closed curve which is composed of finitely many horizontal line segments and the same number of vertical line segments such that vertical line segments always cross over horizontal line segments. An $n \times n$ *Cromwell matrix* is an $n \times n$ square matrix whose entries are either 0 or 1 such that there are exactly two 1's in each row and each column. By joining the 1's in a Cromwell matrix by horizontal line segments and vertical line segments, we obtain a grid diagram. See Figure 1.



$$\begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Figure 1. A grid diagram and its Cromwell matrix

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On the other hand, a grid diagram can be converted easily to an arc presentation with the number of arcs equal to the number of vertical line segments as illustrated in Figure 2. Conversely, an arc presentation can be easily converted to a grid diagram with the number of vertical line segments equal to the number of arcs.

TABLE 1. Prime knots up to arc index 12 or crossing number 13

Arc index / Crossings	5	6	7	8	9	10	11	12	13	14	15	Total
3	1											1
4		1										1
5			2									2
6				3								3
7					7							7
8			1	2		18						21
9				2	6		41					49
10				1	9	32		123				165
11					4	46	135		367			552
12					2	48	211	627		1288		2176
13						49	399	**1412**	**3250**		4878	9988
14						17	477	**3180**				
15					1	22	441	**6216**				
16						7	345	**7955**				
17						1	192	10283				
18							*75*	*8584*				
19							*12*	*6063*				
20							*3*	*3540*				
21							*3*	*1284*				
22								*761*				
23								*124*				
24							1	*132*				
25								*39*				
26								*3*				
27												
28								1				
Total	1	1	3	8	29	240	2335	*50327*				

For this work, we've generated 12×12 Cromwell matrices so that they cover all prime knots with arc index 12. Using 'Knotscape' [3, 4] we identified the knots corresponding to these matrices. Then we removed those corresponding to the unknot, composite knots, links, and knots with arc index less than 12. The remaining knots have arc index 12. Finally we removed duplicates.

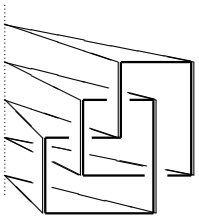
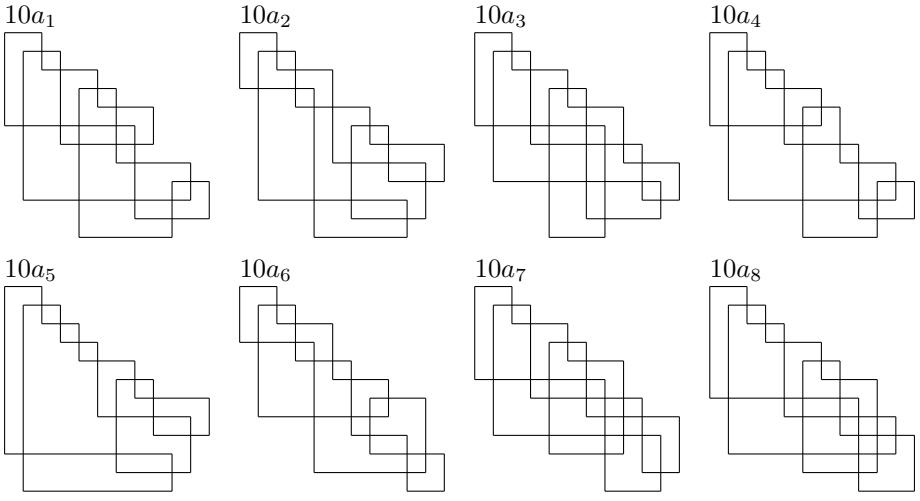


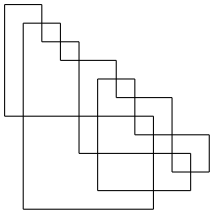
FIGURE 2. Construction of an arc presentation from a grid diagram

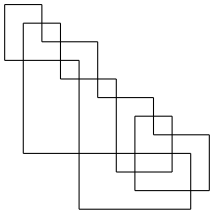
Table 1 shows the number of prime knots of given arc index and minimal crossing number. Minimal grid diagrams of the 282 knots counted for the arc index 5 to 10 appear in [5] and those of the 2,335 knots counted for the arc index 11 appear in [7, 8]. In the columns of the arc index 11 and 12, the knots counted by the thirteen italicized numbers may have smaller crossing numbers but not smaller than 17. The five numbers in boldface are newly determined by this work.

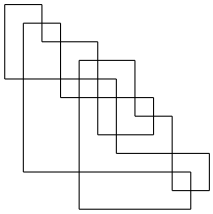
2. Minimal grid diagrams of the prime knots with arc index 12

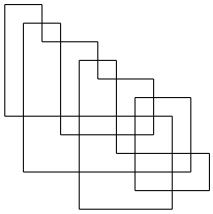
In grid diagrams, we use the convention that the vertical lines cross over the horizontal ones at every crossing. To save pages, we provide minimal grid diagrams only for the prime alternating knots with arc index 12. They are the 123 prime alternating 10 crossing knots. The full list of 19,513 prime knots of arc index 12 up to 16 crossings and their minimal grid diagrams can be found in the arXiv [6]. The labels of the grid diagrams are the DT names [9].

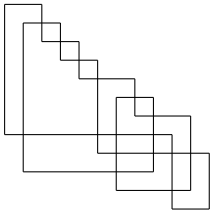


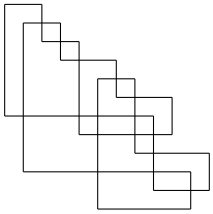
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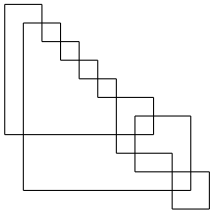
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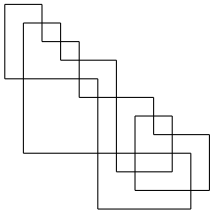
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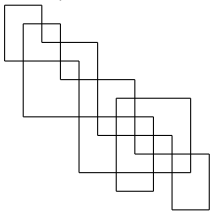
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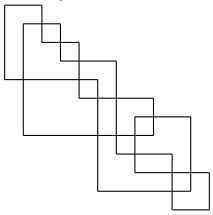
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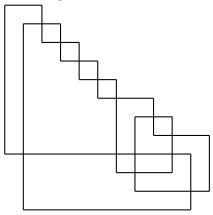
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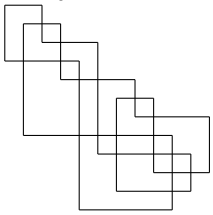
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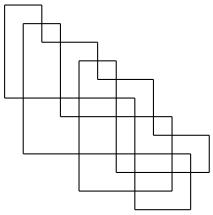
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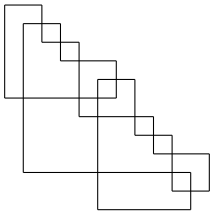
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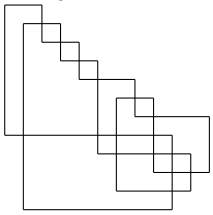
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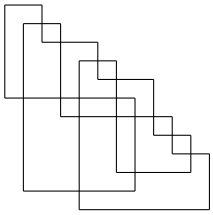
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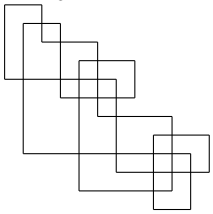
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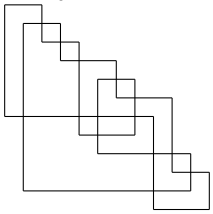
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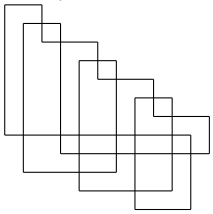
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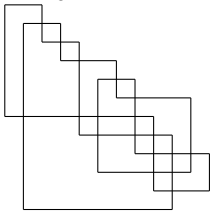
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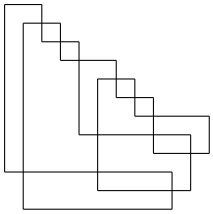
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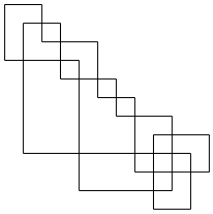
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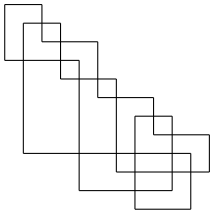
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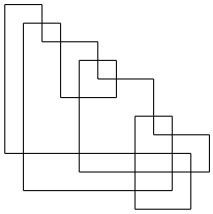
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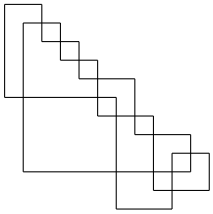
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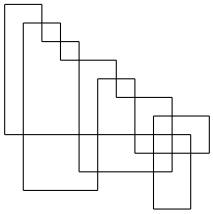
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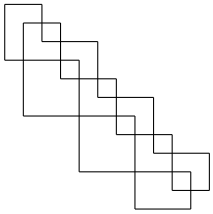
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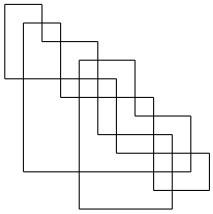
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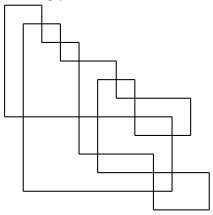
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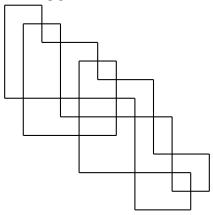
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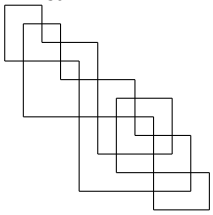
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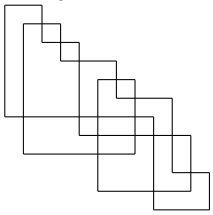
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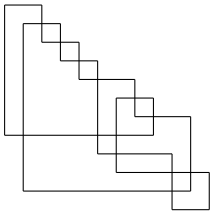
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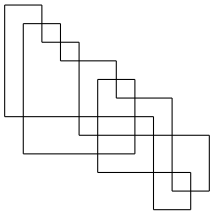
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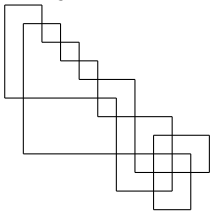
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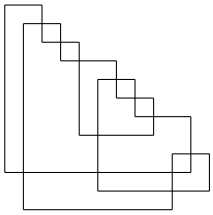
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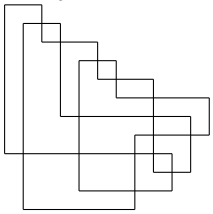
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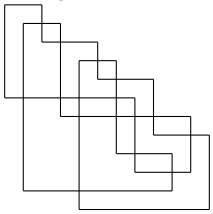
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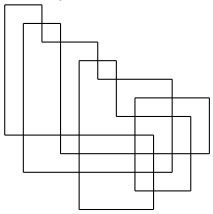
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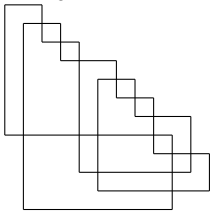
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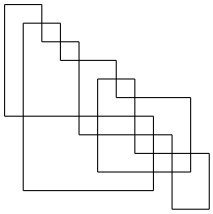
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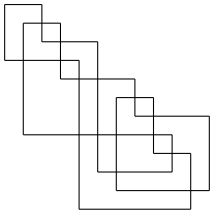
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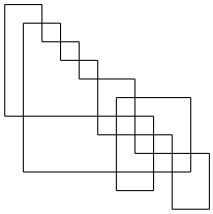
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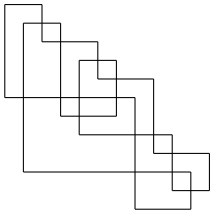
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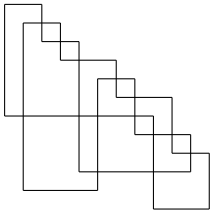
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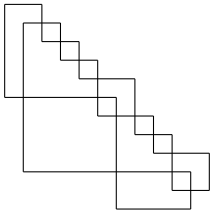
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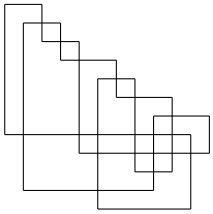
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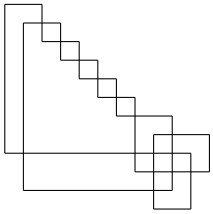
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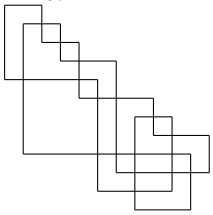
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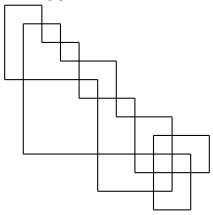
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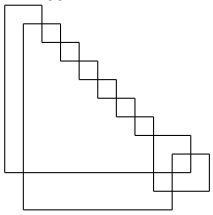
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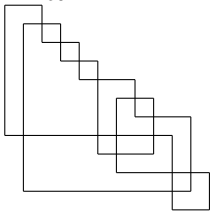
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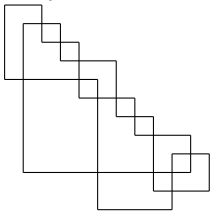
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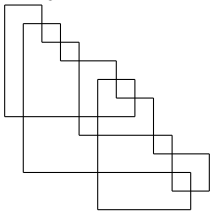
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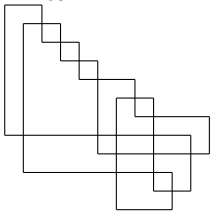
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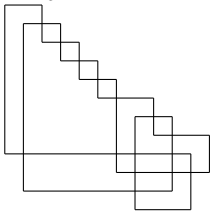
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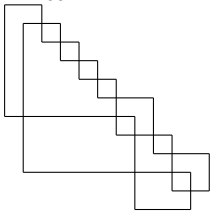
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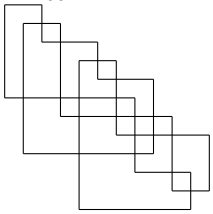
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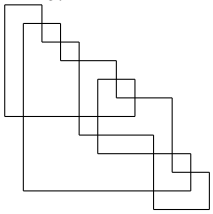
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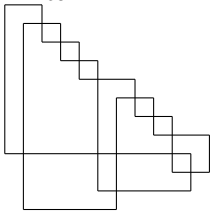
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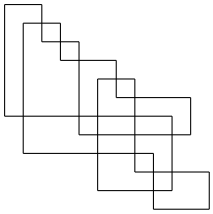
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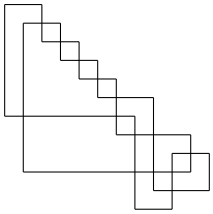
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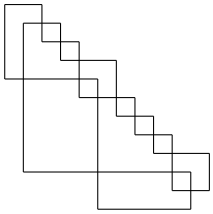
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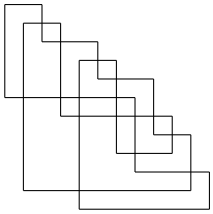
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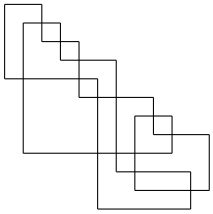
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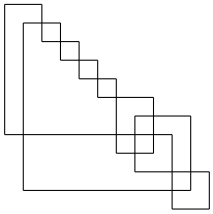
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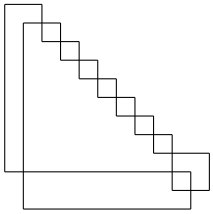
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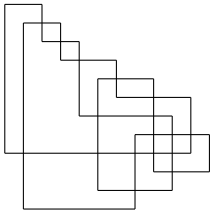
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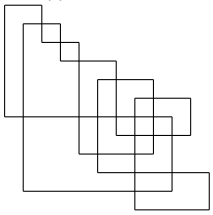
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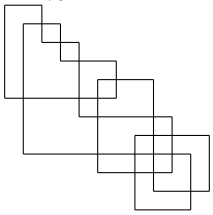
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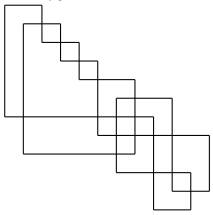
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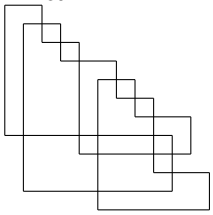
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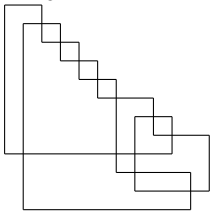
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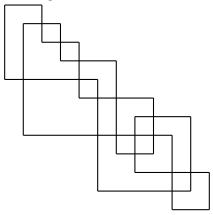
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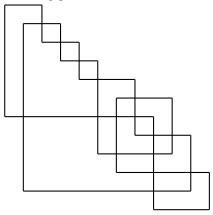
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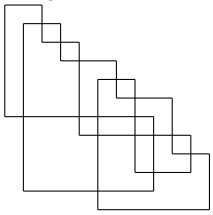
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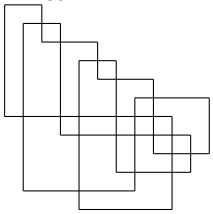
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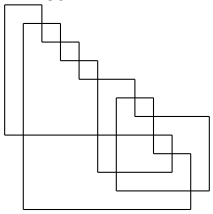
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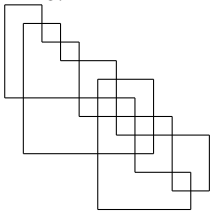
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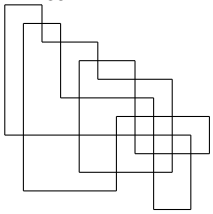
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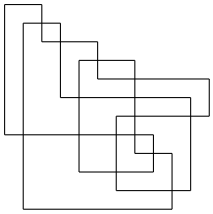
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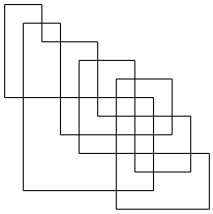
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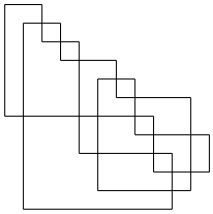
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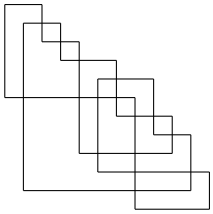
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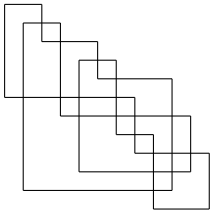
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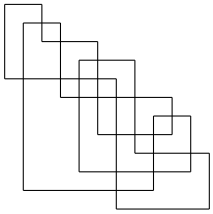
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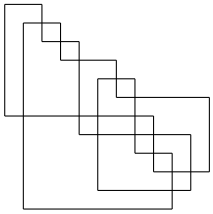
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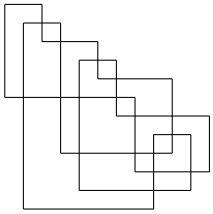
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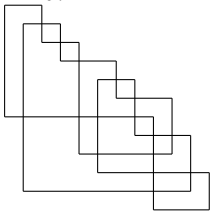
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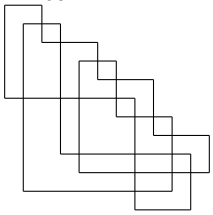
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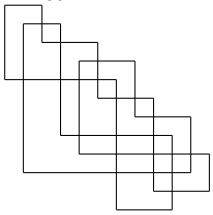
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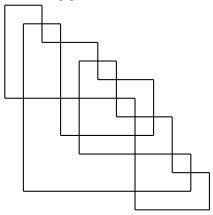
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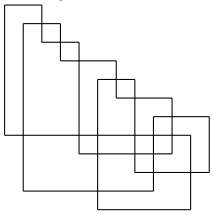
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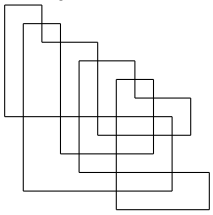
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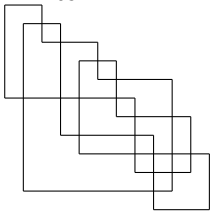
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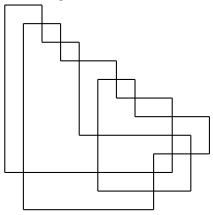
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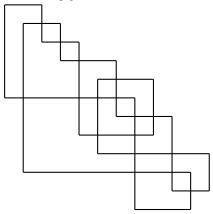
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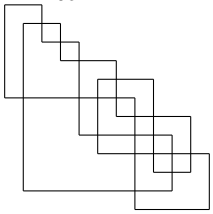
$10a_{101}$ 

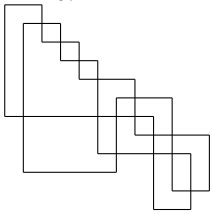
$10a_{102}$ 

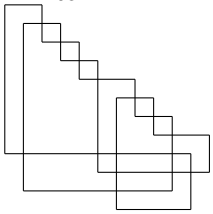
$10a_{103}$ 

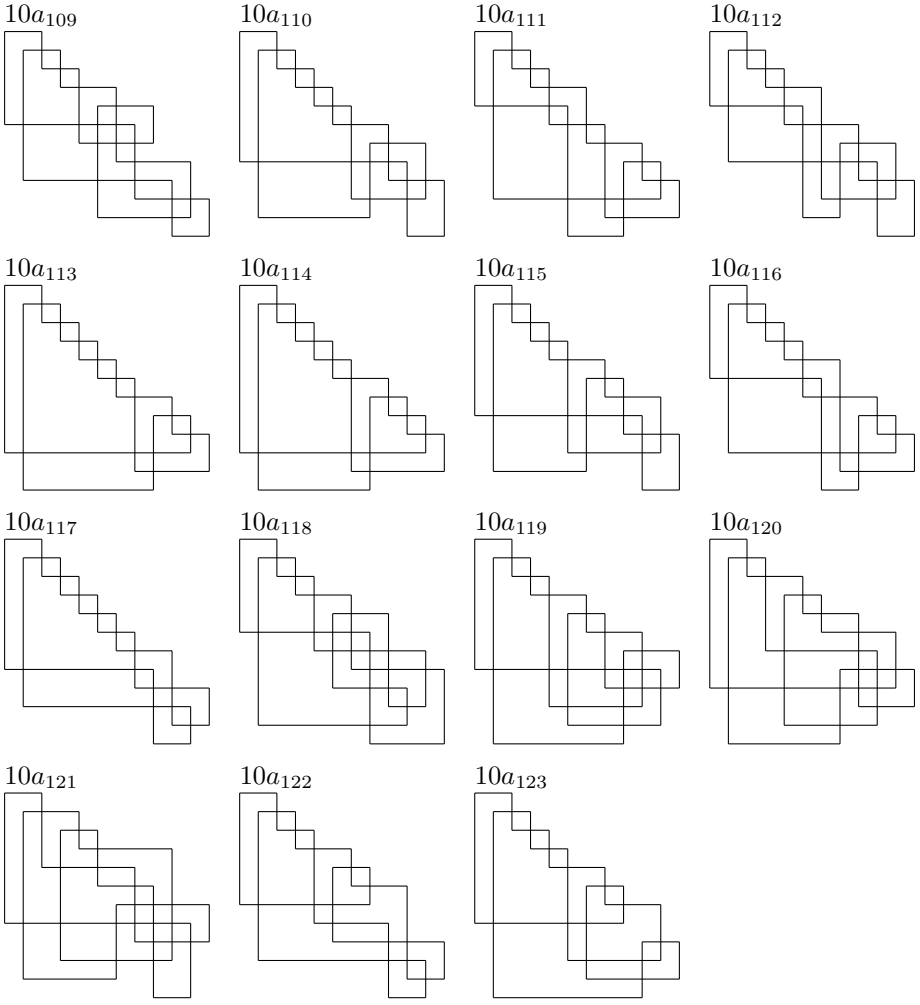
$10a_{104}$ 

$10a_{105}$ 

$10a_{106}$ 

$10a_{107}$ 

$10a_{108}$ 



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